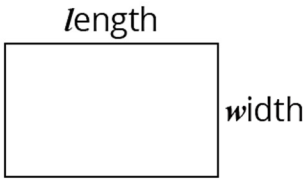
GCSE Maths **Formulae** (Higher)

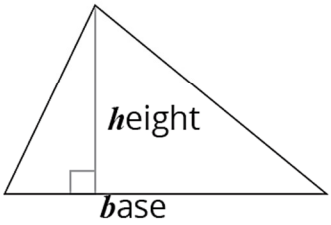
Area of a Rectangle



$length \times width$

$= lw$

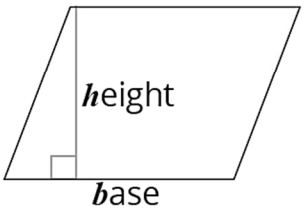
Area of a Triangle



$\frac{1}{2} \times base \times height$

$= \frac{1}{2}bh$

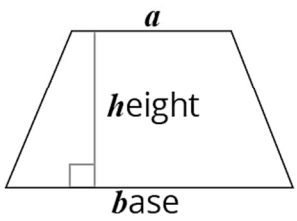
Area of a Parallelogram



$base \times height$

$= bh$

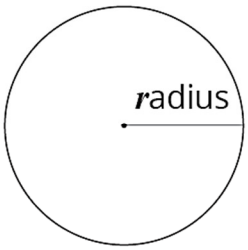
Area of a Trapezium



$\frac{1}{2} \times (a + b) \times height$

$= \frac{1}{2}(a + b)h$

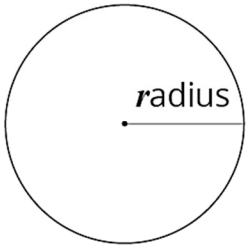
Area of a Circle



$\pi \times radius \times radius$

$= \pi r^2$

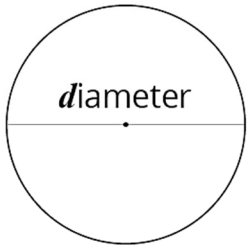
Circumference of a Circle



$2 \times \pi \times radius$

$= 2\pi r$

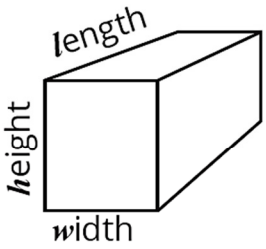
Circumference of a Circle



$\pi \times diameter$

$= \pi d$

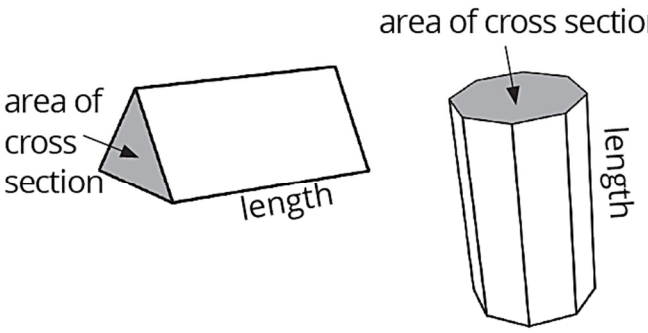
Volume of a Cuboid



$length \times width \times height$

$= lwh$

Volume of a Prism



$area\ of\ cross\ section \times length$

Volume of a Sphere

(Given in relevant questions)

$\frac{4}{3} \times \pi \times radius \times radius \times radius$

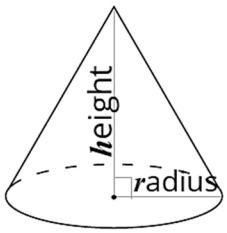
$= \frac{4}{3}\pi r^3$

Volume of a Cone

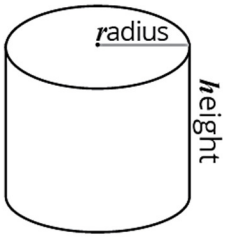
(Given in relevant questions)

$\frac{1}{3} \times \pi \times radius \times radius \times height$

$= \frac{1}{3}\pi r^2 h$



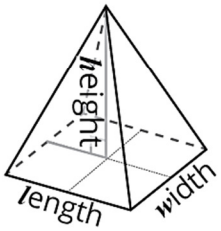
Volume of a Cylinder



$\pi \times radius \times radius \times height$

$= \pi r^2 h$

Volume of a Rectangular Based Pyramid

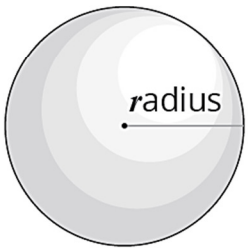


$\frac{1}{3} \times length \times width \times height$

$= \frac{1}{3}lwh$

Surface Area of a Sphere

(Given in relevant questions)

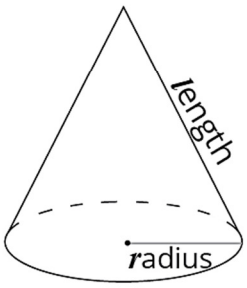


$4 \times \pi \times radius \times radius$

$= 4\pi r^2$

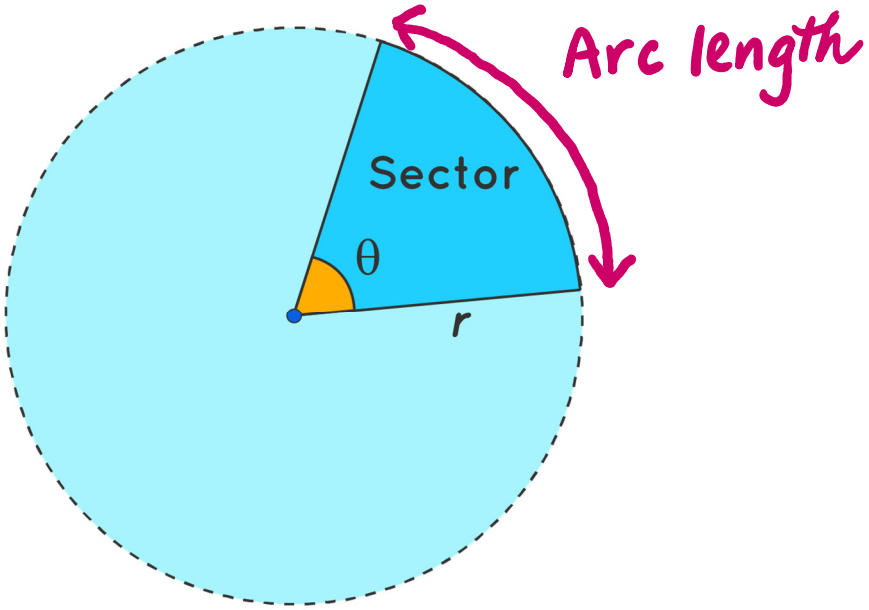
Curved Surface Area of a Cone

(Given in relevant questions)



$\pi \times radius \times length$

$= \pi rl$



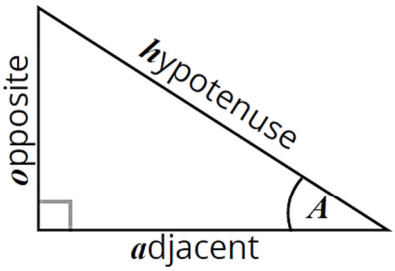
Area of sector:

$$\pi r^2 \times \frac{\theta}{360}$$

Arc Length:

$$\pi d \times \frac{\theta}{360}$$

Trigonometry Formulae



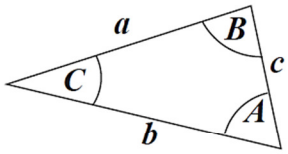
$\text{Sin } A = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{Cos } A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{Tan } A = \frac{\text{opposite}}{\text{adjacent}}$

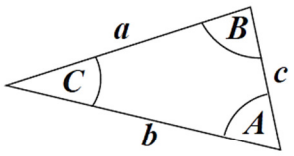
$\text{Sin } A = \frac{o}{h}$, $\text{Cos } A = \frac{a}{h}$, $\text{Tan } A = \frac{o}{a}$

Sine Rule



$$\frac{a}{\text{Sin}A} = \frac{b}{\text{Sin}B} = \frac{c}{\text{Sin}C}$$

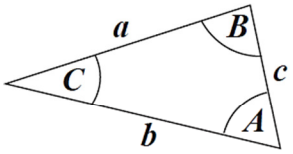
Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\text{Cos}A$$

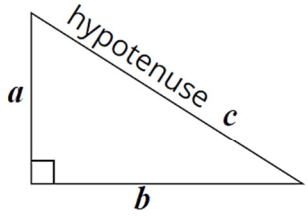
$$\text{cosA} = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of ANY Triangle



$$\frac{1}{2}ab\text{Sin}C$$

Pythagoras' Theorem



$$a^2 + b^2 = c^2$$

Values of Trigonometric Functions

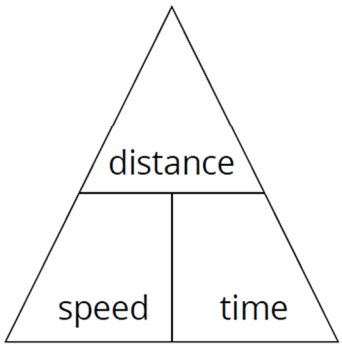
	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

Quadratic Formula

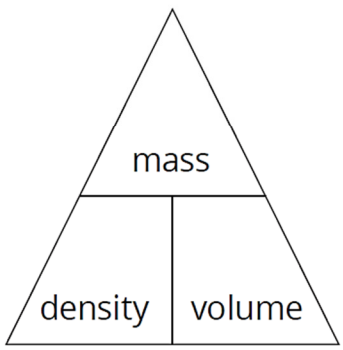
For: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

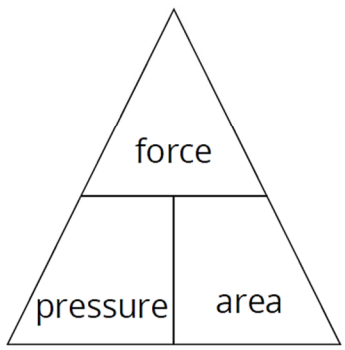
Compound Measures: Speed



Compound Measures: Density



Compound Measures: Pressure



Probability

P(A) is Probability of outcome A

P(B) is Probability of outcome B

P(A or **B)** = **P(A)** + **P(B)** − **P(A** and **B)**

P(A and **B)** = **P(A** given **B)P(B)**

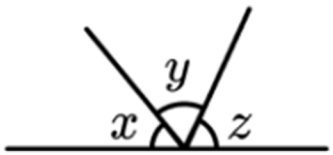
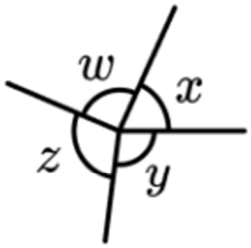
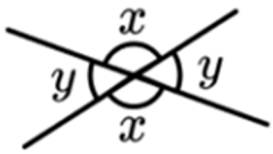
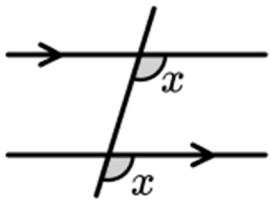
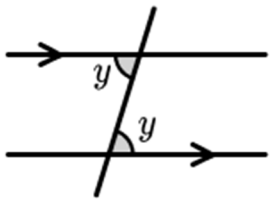
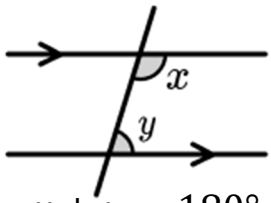
Compound Interest

Principle amount

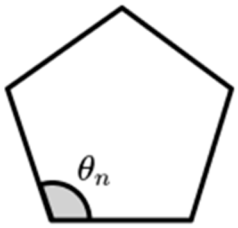
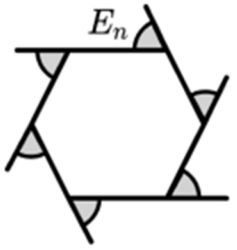
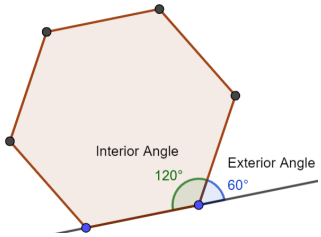
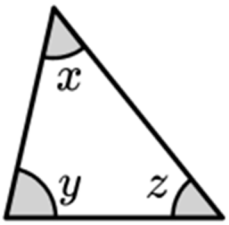
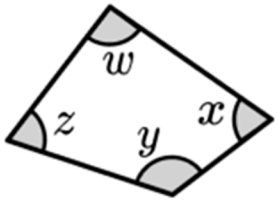
interest **r**ate

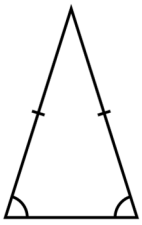
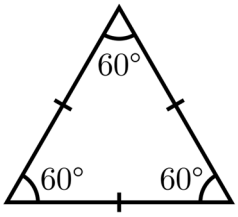
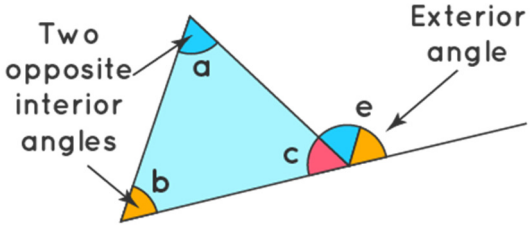
number of times the interest is compounded

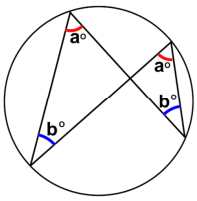
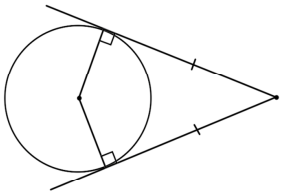
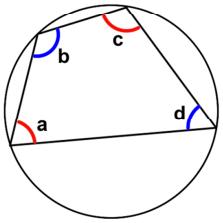
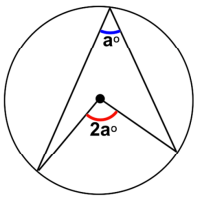
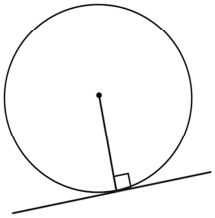
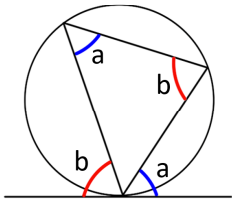
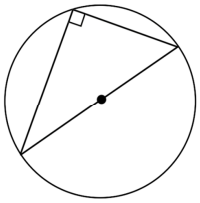
$$\text{Value of Investment} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100}\right)^{\mathbf{n}}$$

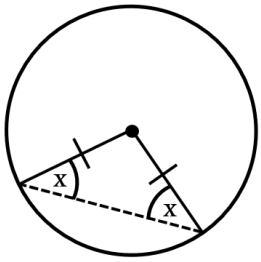
Angle Rule	Description	Diagram
Angles on a straight line	Angles on a straight line add up to 180° $x + y + z = 180$	
Angles at a point	Angles at a point add up to 360° $w + x + y + z = 360$	
Vertically opposite angles	Vertically opposite angles are equal.	
Corresponding angles	Corresponding angles are equal	
Alternate angles	Alternate angles are equal	
Co-interior angles	Co-interior angles add up to 180°	 $x + y = 180°$

Type of quadrilateral	Angle property
Square / Rectangle	All four angles are equal to 90°
Parallelogram / Rhombus	Two pairs of opposite angles are equal
Kite / Arrowhead	One pair of equal angles
Trapezium	Two pairs of co-interior angles (see co-interior angles in parallel lines below)

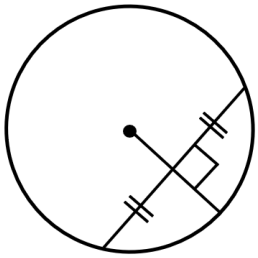
Sum of Interior Angles for an n-sided polygon $(n-2)\times 180°$ For a regular polygon, the size of each interior angle is $\frac{(n-2)\times 180°}{n}$		
Exterior angles of polygons	Exterior angles of a polygon add up to $360°$	
Interior and Exterior	The interior and exterior angle of any polygon add up to $180°$	
Angles in a triangle	Angles in a triangle add up to $180°$	
Angles in a quadrilateral	Angles in a quadrilateral add up to $360°$	

Base Angles of an isosceles triangle are equal.	
Angles in an equilateral triangle are equal.	
An exterior angle (of a triangle) is equal to the sum of the internal opposite angles.	

Angles	Tangent	Opposite/ Alternate
Angles in the **same segment** are **equal**. 1 	**Tangents** from an **external point** are **equal** in length 4 	**Opposite angles** of a **cyclic quadrilateral** add to **180°** $a + c = 180°$ $b + d = 180°$ 6 
Angle at the **centre of a circle** is **twice** the angle at the **circumference**. 2 	**Tangents** to a circle is **perpendicular** (90°) to the **radius**. 5 	**Alternate segment theorem** 7 
Angles in a **semicircle** are $\mathbf{90°}$ 3 		



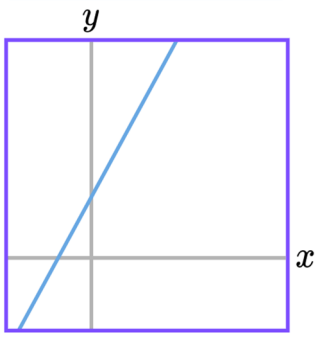
Two radii make an isosceles triangle



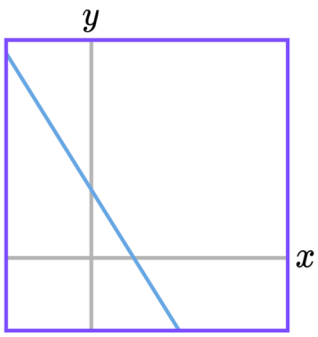
The perpendicular bisector of a chord passes through the centre of the circle

Straight line graphs $y = mx + c$

m is positive



m is negative

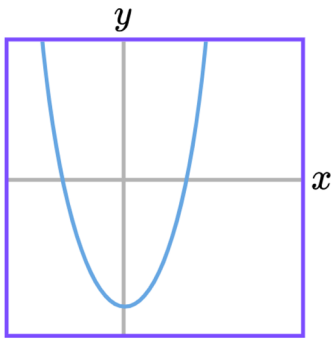


Gradient of parallel lines: Same

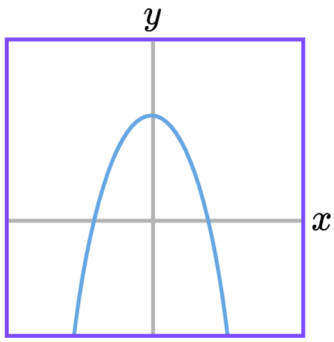
Gradient of perpendicular lines:
$m_1 \times m_2 = -1$
(Negative reciprocal)

Quadratic graphs $y = ax^2 + bx + c$

x^2 term is positive

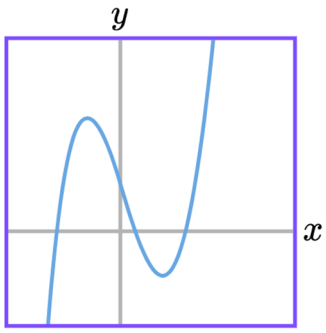


x^2 term is negative

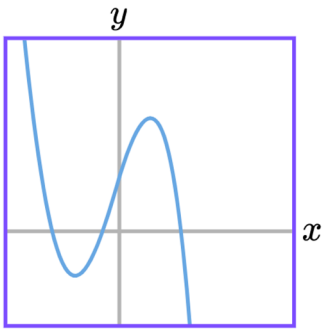


Cubic Graphs $y = ax^3$

x^3 term is positive

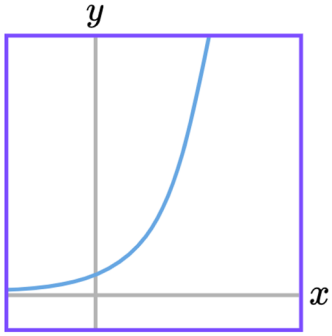


x^3 term is negative

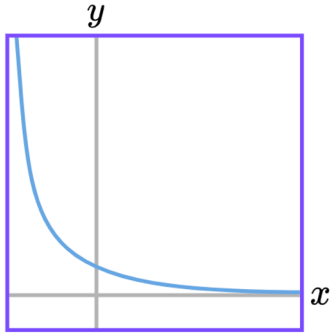


Exponential Graphs $y = k^x$

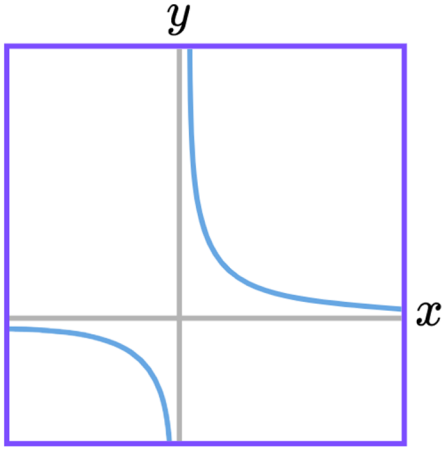
A growth curve



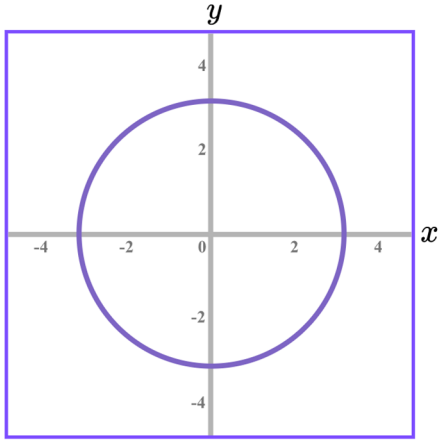
A decay curve



Reciprocal Graphs $y = \frac{1}{x}$



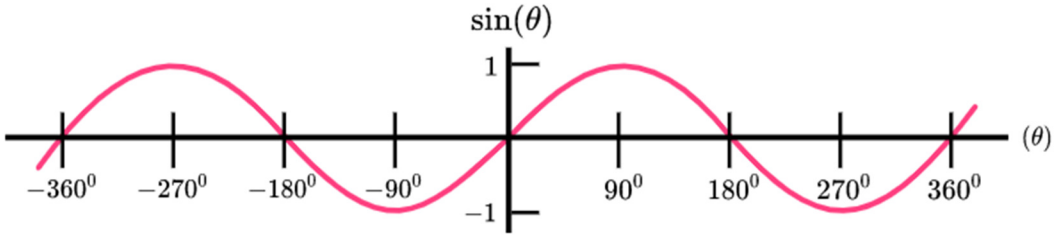
Circle Graphs $x^2 + y^2 = r^2$



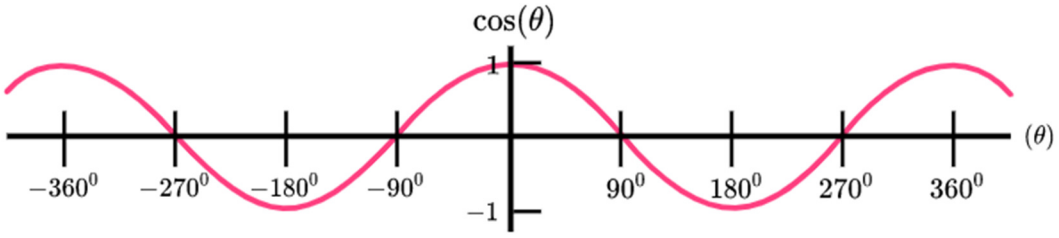
Its equation is:

$x^2 + y^2 = 3^2$

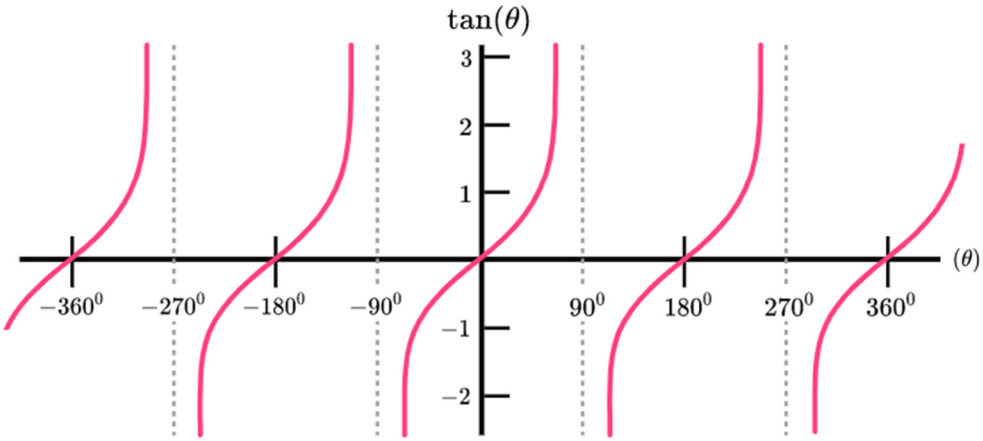
The graph of $y = sin(\theta)$



The graph of $y = cos(\theta)$



The graph of $y = tan(\theta)$



Graphs Transformation

$y = f(x) + a$ $= \begin{pmatrix} 0 \\ a \end{pmatrix}$	$y = f(x - a)$ $= \begin{pmatrix} a \\ 0 \end{pmatrix}$
$y = -f(x)$ Reflection in x axis.	$y = f(-x)$ Reflection in y axis.

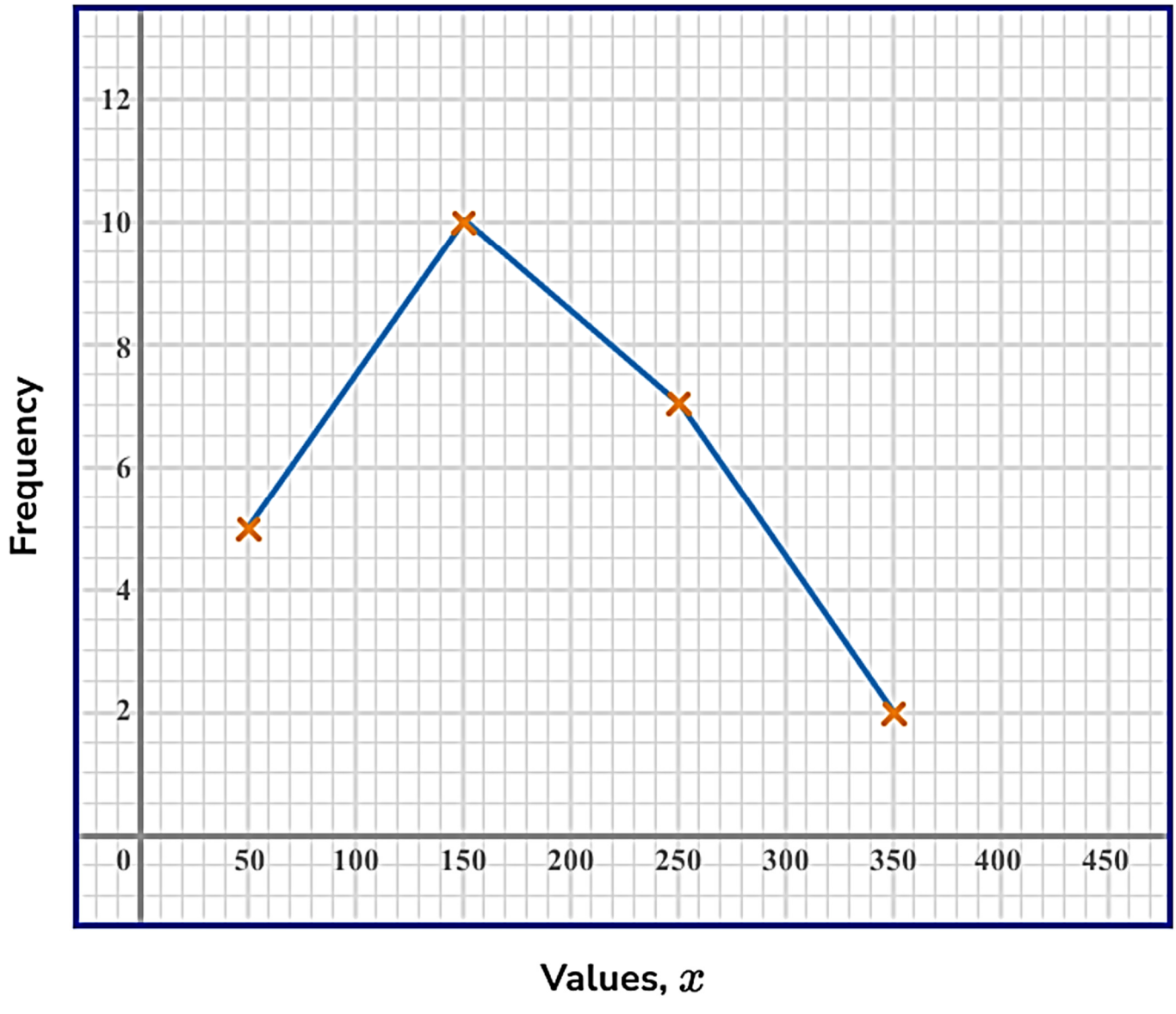
Combined transformation:
The graph of $y = -f(-x)$ are equivalent to a rotation of 180° about the origin.

What is a frequency polygon? Midpoint

A **frequency polygon** is a graph that shows the frequencies of grouped data. It is a type of frequency diagram that plots the **midpoints** of the **class intervals** against the frequencies and then joins up the points with straight lines.

Below is an example of a frequency polygon, with the associated data table.

Values, x	Frequency
$0 \leq x < 100$	5
$100 \leq x < 200$	10
$200 \leq x < 300$	7
$300 \leq x < 400$	2



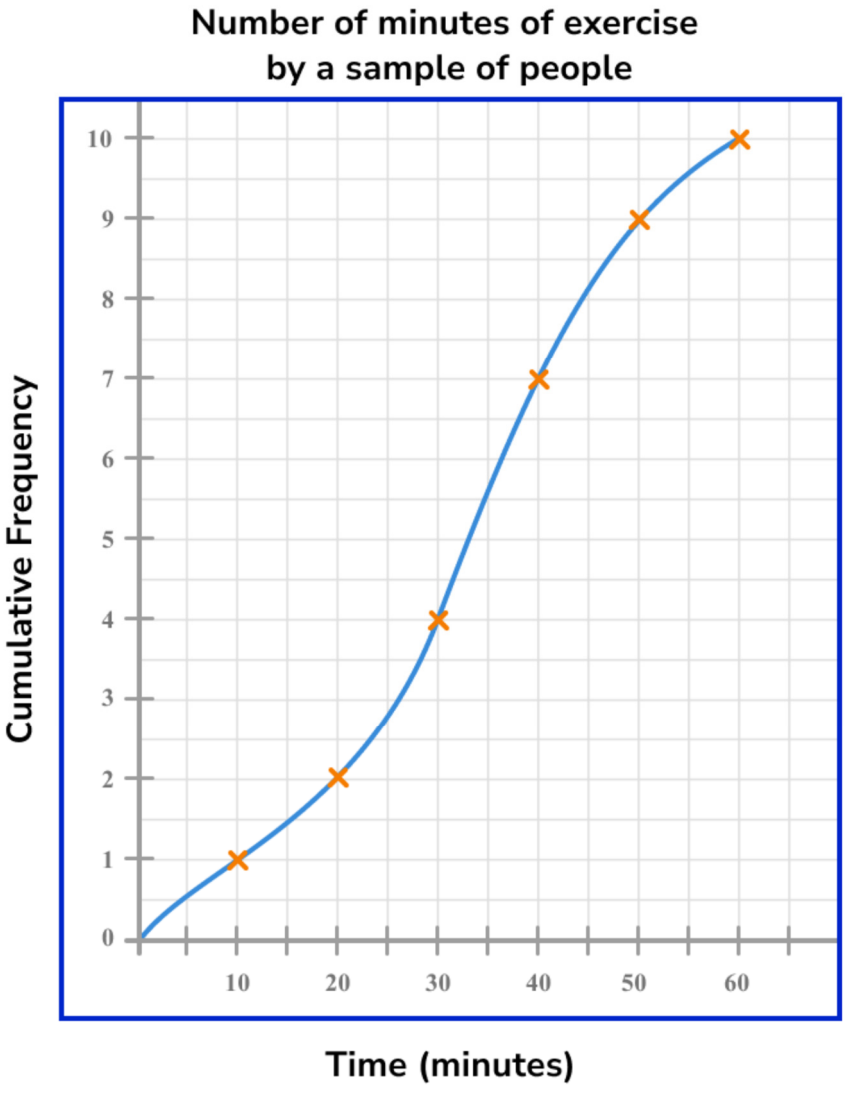
What is cumulative frequency? Cumulative : up to

Cumulative frequency is the running total of frequencies in a frequency distribution.

The horizontal axis of a cumulative frequency graph is marked with the class intervals from the data set to be plotted on a continuous scale. Data points are plotted on the upper class boundary.

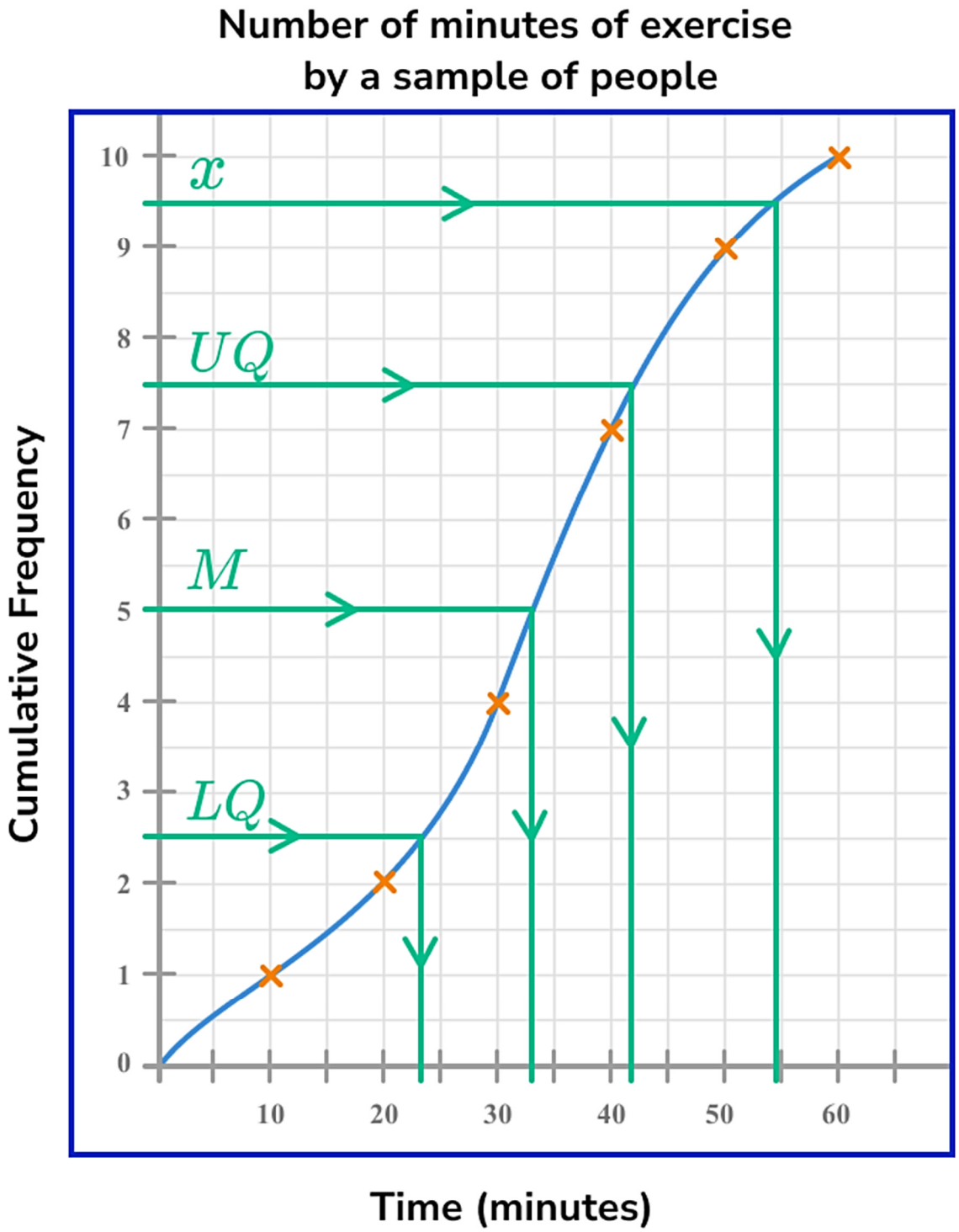
The vertical axis of a cumulative frequency graph is always labelled cumulative frequency.

Time (minutes)	Frequency
$0 < t \leq 10$	1
$10 < t \leq 20$	1
$20 < t \leq 30$	2
$30 < t \leq 40$	3
$40 < t \leq 50$	2
$50 < t \leq 60$	1



Reading data from a cumulative frequency graph

Value	Percentage of data below this value
Lower Quartile (LQ or Q1)	25% of the data lies below this value
Median (M or Q2)	50% of the data lies below this value
Upper Quartile (UQ or Q3)	75% of the data lies below this value
x^{th} Percentile	x% of the data lies below this value



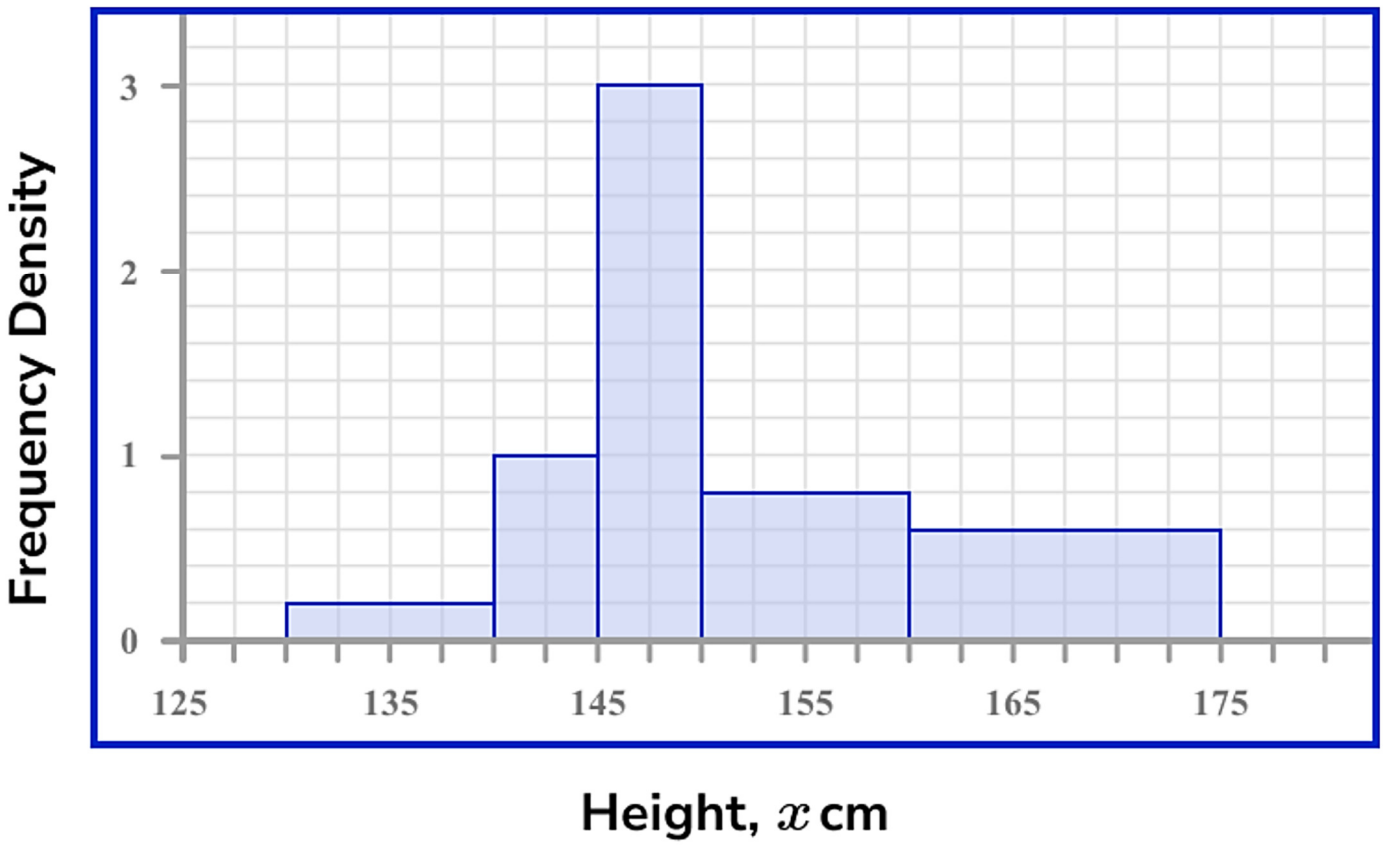
What is a histogram?

A **histogram** is similar to a bar chart but is used to display quantitative **continuous data (numeric data)**, whereas a bar chart (or bar graph) is used to display qualitative or quantitative **discrete data**.

Below is a grouped frequency table and the associated histogram.

Area = height × width

Height, cm	Frequency	Frequency Density	Class width
$130 \leq x < 140$	2	0.2	× 10
$140 \leq x < 145$	5	1	× 5
$145 \leq x < 150$	15	3	× 5
$150 \leq x < 160$	8	0.8	× 10
$160 \leq x < 175$	9	0.6	× 15



Inverse Functions:

Step 1: let $f(x)$ be y

Step 2: make x the subject

Step 3: swap x and y

Find $f^{-1}(x)$

Q1) $f(x) = \frac{-6x+2}{5x-4}$

$$y = \frac{-6x+2}{5x-4}$$

$$y(5x-4) = -6x+2$$

$$5yx - 4y = -6x + 2$$

$$5yx + 6x = 2 + 4y$$

$$x(5y+6) = 2+4y$$

$$x = \frac{2+4y}{5y+6}$$

$$y = \frac{2+4x}{5x+6}$$

$$f^{-1}(x) = \frac{2+4x}{5x+6}$$